THE VISCOSITY OF A PLASMA IN A STRONG MAGNETIC FIELD

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The viscosity of a plasma is studied under conditions in which a magnetic field influences particle collisions. The expressions obtained for the viscosity coefficients differ significantly from those obtained in the normal theory. It is shown that in sufficiently strong magnetic fields a temperature difference arises between the electron and ion plasma components which is proportional to the drift velocity and depends logarithmically on the magnetic field strength.

1. We shall consider a plasma flux in a uniform constant magnetic field H. If the average mass flow velocity v_0 is a function of the coordinates, then stresses will arise caused by the transport of particle momentum.

Knowing the velocity distribution function of plasma particles of type α, $f_\alpha(\mathbf{r}, v_\alpha, t)$, we can find pressure tensor:

$$p_{ij}^{(\alpha)} = \int m_\alpha (\mathbf{v}_\alpha - \mathbf{v}_0)_i (\mathbf{v}_\alpha - \mathbf{v}_0)_j f_\alpha (\mathbf{r}, \mathbf{v}_\alpha t)\, d\mathbf{v}_\alpha . \quad (1.1)$$

The function f_α is the solution of the kinetic equation:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}_\alpha \frac{\partial f_\alpha}{\partial \mathbf{r}} + \Omega_\alpha (\mathbf{h} \times \mathbf{v}_\alpha) \frac{\partial f_\alpha}{\partial \mathbf{v}_\alpha} = I_{st}, \qquad \Omega_\alpha = \frac{q_\alpha H}{m_\alpha c} . \quad (1.2)$$

Here I_{st} is a term describing particle collisions; Ω_α is the Larmor frequency; **h** is a unit vector directed along the magnetic field.

We shall consider that f_α does not differ radically from the Maxwell function and may be represented in the form

$$f_\alpha = f_\alpha^{(0)} (1 + \Phi_\alpha) \qquad (\Phi_\alpha \ll 1) . \quad (1.3)$$

Here $f_\alpha^{(0)}$ is the "local" Maxwell function:

$$f_\alpha^{(0)} = n_\alpha (\mathbf{r}, t) \left(\frac{m_\alpha}{2\pi T(\mathbf{r}, t)}\right)^{3/2} \times$$
$$\times \exp\left\{-\frac{m_\alpha}{2T(\mathbf{r}, t)} [\mathbf{v}_\alpha - \mathbf{v}_0 (\mathbf{r}, t)]^2\right\} . \quad (1.4)$$

Then, with account for the fact that $\Phi_\alpha \ll 1$, the kinetic equation (1.2) can be linearized.

Passing to a system of coordinates moving with velocity v_0, we can reduce this equation to the following form after fairly straightforward transformations:

$$\frac{m_\alpha}{2T} v_{\alpha i} v_{\alpha j} e_{ij}^\circ f_\alpha^{(0)} = -\Omega_\alpha (\mathbf{v}_\alpha \times \mathbf{h}) \frac{\partial \Phi_\alpha}{\partial \mathbf{v}_\alpha} \cdot f_\alpha^{(0)} + I_{st}(\Phi) \quad (1.5)$$

$$e_{ij}^\circ = \frac{\partial v_{0i}}{\partial x_i} + \frac{\partial v_{0j}}{\partial x_i} - \frac{2}{3} \delta_{ij} \operatorname{div} \mathbf{v}_0 . \quad (1.6)$$

Here $I_{st}(\Phi)$ is the part of the collision integral linear in Φ, e_{ij}° is the drift velocity tensor.

In all preceding papers on the theory of viscosity coefficients, either the classical Boltzmann collision integral (see, e.g., [1-3]), or the Landau collision integral (see, e.g., [4])[1] has been used for $I_{st}(\Phi)$. Neither of these forms of the collision integral is applicable under conditions when the Larmor radii of the particles become less than the radii of their effective interaction region.

Recently, a series of collision integrals [7-9] was obtained which enables one to consider transport processes in fairly strong magnetic fields when the Larmor radii of the particles may be less than the dimensions of their effective region of interaction.

We shall employ the following form of the collision integral (see [10])

$$I_{st}(\Phi) = \frac{\partial}{\partial \mathbf{v}_\alpha} \sum_{\beta=e,i} \mathbf{J}_{\alpha\beta} , \quad (1.7)$$

where

$$\mathbf{J}_{\alpha\beta} = \frac{2}{\pi m_\alpha} (q_\alpha q_\beta)^2 \int \frac{dk}{k^4} \mathbf{k} \int_0^{\tau_{max}} d\tau S_\tau^{(0)} \mathbf{k} \left(\frac{\partial}{\partial \mathbf{p}_\alpha} - \frac{\partial}{\partial \mathbf{p}_\beta}\right) \times$$
$$\times f_\alpha f_\beta \exp [i\mathbf{k} (S_\tau^{(0)} - 1)(\mathbf{r} - \mathbf{r}_\beta)]\, d\mathbf{v}_\beta .$$

Here τ_{max} is the maximum particle interaction time, $S_\tau^{(0)}$ is an operator which replaces the dynamic variables of particles situated in a homogeneous magnetic field by their values after a time τ, assuming the particles to be noninteracting.

2. We shall seek a solution of the kinetic equation (1.5) in the form

$$\Phi_\alpha = \Phi_{ij}^\alpha w_{\alpha i} w_{\alpha j} \qquad (w_\alpha = (2T / m_\alpha)^{1/2} v_\alpha) \quad (2.1)$$

(w_α is the dimensionless velocity).

The tensor Φ_{ij}^α must be linear in the disturbance e_{ij}°. It is easy to show that if we have the pseudovector h, we can form only the following six independent mutually orthogonal tensors of the second rank linear in e°:

$$Q_{ij}^{(0)} = h_i h_j h_\mu h_\nu e_{\mu\nu}^\circ, \qquad Q_{ij}^{(3)} = {}^1/_2 (\delta_{i\mu}^\perp \varepsilon_{j\gamma\nu} + \delta_{j\nu}^\perp \varepsilon_{i\gamma\mu}) h_\gamma e_{\mu\nu}^\circ,$$
$$Q_{ij}^{(1)} = \left(\delta_{i\mu}^\perp \delta_{j\nu}^\perp + {}^1/_2 \delta_{ij}^\perp h_\mu h_\nu\right) e_{\mu\nu}^\circ,$$
$$Q_{ij}^{(4)} = (h_i h_\mu \varepsilon_{j\gamma\nu} + h_j h_\nu \varepsilon_{i\gamma\mu}) h_\gamma e_{\mu\nu}^\circ ,$$
$$Q_{ij}^{(2)} = (\delta_{i\mu}^\perp h_j h_\nu + \delta_{j\nu}^\perp h_i h_\mu) e_{\mu\nu}^\circ,$$
$$Q_{ij}^{(5)} = \delta_{ij}^\perp h_\mu h_\nu e_{\mu\nu}^\circ \quad (\delta_{ij}^\perp = \delta_{ij} - h_i h_j) . \quad (2.2)$$

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[1] We are talking about collision integrals for pair interactions. The influence of collective effects on transport phenomena was considered in [5, 6].

Here $\varepsilon_{ij\gamma}$ is an antisymmetric tensor. All these tensors, with the exception of $Q^{(0)}$ and $Q^{(5)}$, are nondivergent, i.e., they have a trace equal to zero. Thus, $\Phi_{ij}^{(\alpha)}$ is a linear combination of these six tensors:

$$\Phi_\alpha = -\sum_{p=0}^{5} b_\alpha^{(p)} Q_{ij}^{(p)} w_{\alpha i} w_{\alpha j}. \qquad (2.3)$$

The coefficients $b_\alpha^{(p)}$ are, generally speaking, functions of w_α^2. We shall, however, consider them independent of velocity, which corresponds to the first approximation in the expansion of these coefficients in a series of Laguerre polynomials.

Setting (2.3) in (1.5), we obtain, after multiplying by $w_{\alpha i} w_{\alpha j} dv_\alpha$ and integrating,

$$e_{ij}^\circ = \Omega_\alpha (2b_\alpha^{(1)} Q_{ij}^{(3)} + b_\alpha^{(2)} Q_{ij}^{(4)} - 2b_\alpha^{(3)} Q_{ij}^{(1)} - b_\alpha^{(4)} Q_{ij}^{(2)}) +$$
$$+ \frac{2}{\pi^4 m_\alpha T} \sum_{\beta=e,i} (q_\alpha q_\beta)^2 n_\beta \int \frac{dk}{k^4} k_t \int_0^{\tau_{max}} d\tau \int dw_\alpha dw_\beta \times$$
$$\times \exp[-w_\alpha^2 - w_\beta^2 + i\mathbf{k}(S_\tau^{(0)} - 1)(\mathbf{r} - \mathbf{r}_\beta)] \times$$
$$\times (w_{\alpha i} k_j + w_{\alpha j} k_i) \sum_{p=0}^{5} Q_{rl}^{(p)} S_\tau^{(0)} \times$$
$$\times \left[b_\alpha^{(p)} w_{\alpha r} - \left(\frac{m_\alpha}{m_\beta}\right)^{1/2} b_\beta^{(p)} w_{\beta r} \right]. \qquad (2.4)$$

We represent the action of the operator $S_\tau^{(0)}$ on the dynamic variables in the form:

$$S_\tau^{(0)} w_{\alpha i} = (h_i h_m + \sin \Omega_\alpha \tau \varepsilon_{imk} h_k + \cos \Omega_\alpha \tau \delta_{im}{}^{\perp}) w_{\alpha m}, \quad (2.5)$$

$$S_\tau^{(0)} r_{\alpha i} = \int_0^\tau S_{\tau'}^{(0)} w_{\alpha i} d\tau'. \qquad (2.6)$$

After this, we carry out the integration over velocities in the collision term in (2.4). Writing e° in the form

$$e^\circ = Q^{(0)} + Q^{(1)} + Q^{(2)} - {}^1/_2 Q^{(5)} \qquad (2.7)$$

and using the orthogonality property of the tensors $Q^{(p)}$, we obtain

$$1 = \frac{2}{\pi m_\alpha T} \sum_{\beta=e,i} (q_\alpha q_\beta)^2 n_\beta \{ b_\alpha^{(0)} [-L_1^{\alpha\beta} (\rho_\alpha^2 \Omega_\alpha^2 \tau^2) + 2K_2^{\alpha\beta}] -$$
$$- 2b_\alpha^{(5)} L_3^{\alpha\beta} (\rho_\alpha^2 \Omega_\alpha \tau \sin \Omega_\alpha \tau) - \qquad (2.8)$$
$$- \rho_\alpha^2 \frac{q_\alpha}{q_\beta} [b_\beta^{(0)} L_1^{\alpha\beta} (\Omega_\alpha \Omega_\beta \tau^2) + 2b_\beta^{(5)} L_3^{\alpha\beta} (\Omega_\alpha \tau \sin \Omega_\beta \tau)] \},$$

$$-\frac{1}{2} = \frac{2}{\pi m_\alpha T} \sum_{\beta=e,i} (q_\alpha q_\beta)^2 n_\beta \{ b_\alpha^{(0)} [-L_3^{\alpha\beta} (\rho_\alpha^2 \Omega_\alpha \tau \sin \Omega_\alpha \tau)] +$$
$$+ 2b_\alpha^{(5)} [K_1^{\alpha\beta} (\cos \Omega_\alpha \tau) - 2L_2^{\alpha\beta} (\rho_\alpha^2 \sin^2 \Omega_\alpha \tau)] -$$
$$- \rho_\alpha^2 [b_\beta^{(0)} L_3^{\alpha\beta} (\Omega_\beta \tau \sin \Omega_\alpha \tau) +$$
$$+ 4b_\beta^{(5)} L_2^{\alpha\beta} (\sin \Omega_\alpha \tau \sin \Omega_\beta \tau)] \}, \qquad (2.9)$$

$$1 = -2\Omega_\alpha M_\alpha + \frac{2}{\pi m_\alpha T} \sum_{\beta=e,i} (q_\alpha q_\beta)^2 n_\beta \times$$
$$\times \left\{ -2M_\alpha K_1^{\alpha\beta} \left(\exp\left[i\left(\frac{\pi}{2} \Omega_\alpha \tau\right)\right]\right) + \right.$$
$$+ \frac{8\rho_\alpha^2 q_\alpha}{q_\beta} M_\beta L_2^{\alpha\beta} \left(\sin\frac{\Omega_\alpha \tau}{2} \sin\frac{\Omega_\beta \tau}{2} \exp\left[i\left(\frac{\pi}{2} - \frac{\Omega_\alpha + \Omega_\beta}{2}\tau\right)\right]\right) +$$
$$\left. + 8\rho_\alpha^2 M_\alpha L_2^{\alpha\beta} \left(\sin^2 \frac{\Omega_\alpha \tau}{2} \exp\left[i\left(\frac{\pi}{2} - \Omega_\alpha \tau\right)\right]\right)\right\}, \quad (2.10)$$

$$1 = -\Omega_\alpha N_\alpha + \frac{2}{\pi m_\alpha T} \sum_{\beta=e,i} (q_\alpha q_\beta)^2 n_\beta \left\{ iN_\alpha \left[\rho_\alpha^2 L_3^{\alpha\beta} \left(\frac{\Omega_\alpha^2 \tau^2}{2} + \right.\right.\right.$$
$$\left. + 2\sin^2 \frac{\Omega_\alpha \tau}{2} \cos \Omega_\alpha \tau + \Omega_\alpha \tau \sin \Omega_\alpha \tau \right) - K_1^{\alpha\beta} - K_2^{\alpha\beta} (\cos \Omega_\alpha \tau) \Big] +$$
$$+ N_\alpha \left[2\rho_\alpha^2 L_3^{\alpha\beta} \left(\sin^2 \frac{\Omega_\alpha \tau}{2} \sin \Omega_\alpha \tau + \Omega_\alpha \tau \sin^2 \frac{\Omega_\alpha \tau}{2}\right) - \right.$$
$$\left. - K_2^{\alpha\beta} (\sin \Omega_\alpha \tau)\right] + \frac{\rho_\alpha^2 q_\alpha}{q_\beta} \left[iN_\beta L_3^{\alpha\beta} \left(\frac{\Omega_\alpha \Omega_\beta \tau^2}{2} + 2\sin \frac{\Omega_\alpha \tau}{2} \times \right.\right.$$
$$\left. \times \sin \frac{\Omega_\beta \tau}{2} \cos \frac{\Omega_\alpha + \Omega_\beta}{2} \tau + \frac{\Omega_\alpha \tau}{2} \sin \Omega_\beta \tau + \frac{\Omega_\beta \tau}{2} \sin \Omega_\alpha \tau \right) +$$
$$\left.\left. + 2N_\beta L_3^{\alpha\beta} \left(\sin \frac{\Omega_\alpha \tau}{2} \sin \frac{\Omega_\beta \tau}{2} \sin \frac{\Omega_\alpha + \Omega_\beta}{2} \tau \right)\right]\right\} \qquad (2.11)$$

$$(M_\alpha = b_\alpha^{(3)} + ib_\alpha^{(1)}), \quad N_\alpha = b_\alpha^{(4)} + ib_\alpha^{(2)}).$$

The integral operators $L_i^{\alpha\beta}$ are given by the relations

$$L_1^{\alpha\beta} = \int \frac{d\mathbf{k}}{k^4} k_\parallel^4 \int_0^{\tau_{max}} d\tau e^{-t_\alpha - t_\beta}, \qquad L_2^{\alpha\beta} = \frac{1}{8} \int \frac{dk}{k^4} k_\perp^4 \times$$
$$\times \int_0^{\tau_{max}} d\tau\, e^{-t_\alpha - t_\beta}, \; L_3^{\alpha\beta} = \frac{1}{2} \int \frac{d\mathbf{k}}{k^4} k_\perp^2 k_\parallel^2 \int_0^{\tau_{max}} d\tau\, e^{-t_\alpha - t_\beta}.$$

The integral operators $K_i^{\alpha\beta}$ are given by the relation

$$K_1^{\alpha\beta} = \frac{1}{2} \int \frac{d\mathbf{k}}{k^4} k_\perp^2 \int_0^{\tau_{max}} d\tau\, e^{-t_\alpha - t_\beta},$$

$$K_2^{\alpha\beta} = \int \frac{d\mathbf{k}}{k^4} k_\parallel^2 \int_0^{\tau_{max}} d\tau\, e^{-t_\alpha - t_\beta}$$

$$t_\alpha = \rho_\alpha^2 \left(k_\parallel^2 \frac{\Omega_\alpha^2 \tau^2}{4} + k_\perp^2 \sin^2 \frac{\Omega_\alpha \tau}{2} \right), \quad \rho_\alpha = \frac{1}{\Omega_\alpha} \left(\frac{2T}{m}\right)^{1/2} \quad (2.12)$$

Here ρ_α is the Larmor radius of a type α particle, $k_\parallel$ and $k_\perp$ are components of the vector $\mathbf{k}$ along and across the field, respectively.

The system of equations (2.8)–(2.11) differs from the equations obtained in the usual theory, when the influence of the magnetic field on collisions is not taken into account. The distinctive feature of collision integral (1.7) lies in its explicit dependence on the magnetic field. This dependence manifiests itself in the fact that the operation of the collision integral on any of the $Q^{(p)}$ tensors gives still another tensor, in addition to this one. However, as before, the disturbance may be resolved into three independent parts: $Q^{(0)}$ and $Q^{(5)}$, $Q^{(1)}$ and $Q^{(3)}$, $Q^{(2)}$, and $Q^{(4)}$. In the usual theory Φ_α is sought in the form of an expansion into five, not six tensors (see, for example [4]). In point of fact, this means that only five coefficients in the expansion (2.3) are independent. In the usual theory the coefficients $b_2^{(0)}$ and $b_\alpha^{(5)}$ are connected by the relation

$$b_\alpha^{(0)} + 2b_\alpha^{(5)} = 0 \qquad (2.13)$$

This is obvious if one considers that the viscous stresses arising from a plasma disturbance described by the tensors $Q^{(0)}$ and $Q^{(5)}$ are the same in normal theory as in the case without a magnetic field. As is clear from Eqs. (2.8), (2.9), the magnetic field does not exert a direct influence on these disturbances in the sense that there is no term associated with the Lorentz force. Thus, in theory, with an isotropic collision integral both these disturbances are identical and are described by the one viscosity coefficient. Our collision integral depends explicitly on the magnetic field, and so introduces an additional anisotropy. One may expect the plasma disturbances described by tensors $Q^{(0)}$ and $Q^{(5)}$ to be characterized by different viscosity coefficients. This means different momentum relaxation times along and across the field (cf. [9, 11]), if the form of $Q^{(0)}$ and $Q^{(5)}$ is taken into account.

3. Using the system of equations (2.8)-(2.11), we find a solution of the kinetic equation (1.5) with an accuracy to the function

$$\Phi_\alpha' = a_\alpha^{(1)} + {}^1/_2 a^{(2)} m_\alpha v_\alpha^2 , \tag{3.1}$$

which reduces the right-hand side of (1.5) to zero.

The coefficients $a_\alpha^{(1)}$, $a^{(2)}$ are determined from the following equations, which are a consequence of the fact that the zero-th approximation $f_\alpha^{(0)}$ completely determines the density and temperature of the plasma at a given point

$$\int f_\alpha^{(0)} (\Phi_\alpha + \Phi_\alpha') \, d\mathbf{v}_\alpha = \sum_{\beta=e,i} \int f_\alpha^{(0)} (\Phi_\beta + \Phi_\beta') \frac{m_\beta v_\beta^2}{2} \, d\mathbf{v}_\beta = 0. \tag{3.2}$$

It is not difficult to verify that these requirements are satisfied if

$$a^{(2)} = 0, \qquad a_\alpha^{(1)} = {}^1/_2 (b_\alpha^{(0)} + 2b_\alpha^{(5)}) h_\mu h_\nu e_{\mu\nu}^\circ$$

and we impose the condition

$$\sum_{\beta=e,i} n_\alpha (b_\alpha^{(0)} + 2b_\alpha^{(5)}) = 0 \tag{3.3}$$

on the coefficients $b_\alpha^{(0)}$ and $b_\alpha^{(5)}$.

After this, we can find stress tensor

$$\pi_{ij}^{(\alpha)} = \int f_\alpha^{(0)} (\Phi_\alpha + \Phi_\alpha') m_\alpha v_{\alpha i} v_{\alpha j} d\mathbf{v}_\alpha = \pi_{ij}^{*(\alpha)} + \Delta p_\alpha \delta_{ij} \tag{3.4}$$

from which the nondivergent part has been separated:

$$\pi_{ij}^{*(\alpha)} = - \sum_{p=0}^{4} \eta_\alpha^{(p)} W_{ij}^{(p)}.$$

The second term in (3.4) is a correction to the hydrostatic pressure:

$$\Delta p_\alpha = - {}^1/_3 n_2 T (b_\alpha^{(0)} + 2b_\alpha^{(5)}). \tag{3.5}$$

The viscosity coefficients $\eta_\alpha^{(p)}$ and tensors $W_{ij}^{(p)}$ are determined in the following manner:

$$\eta_\alpha^{(0)} = (b_\alpha^{(0)} - b_\alpha^{(5)}) \eta_\alpha T, \quad \eta_\alpha^{(p)} = \eta_\alpha T b_\alpha^{(p)} \quad \text{for } p \neq 0$$

$$W_{ij}^{(0)} = (h_i h_j - {}^1/_3 \delta_{ij}) h_\mu h_\nu e_{\mu\nu}^\circ, \; W_{ij}^{(p)} = Q_{ij}^{(p)} \quad \text{for } p \neq 0.$$

4. The system of equations (2.8)-(2.11), together with (3.3), allows one to find the viscosity coefficients.

a) Coefficients $\eta_\alpha^{(0)}$. We shall consider the case when the magnetic field is so small that we may neglect effects associated with the finiteness of the Larmor radius. Then, assuming that the magnetic field in (2.8)-(2.9) is equal to zero, we obtain the well-known equations for the viscosity coefficients:

$$\eta_e^{(0)} = \frac{5 n_e T}{4 \nu_e (1 + R) \ln r_D^*}, \qquad \eta_i^{(0)} = \frac{5 n_i T}{4 \nu_i \ln r_D^*},$$

$$R = \frac{1}{z \sqrt{2}}, \qquad \left| \frac{q_i}{q_e} \right| = z, \tag{4.1}$$

$$\nu_e = \frac{4 \sqrt{2\pi} (q_e q_i)^2 n_i}{3 \sqrt{m_e} T^{3/2}}, \qquad \nu_i = \frac{4 \sqrt{\pi} q_i^4 n_i}{3 \sqrt{m_i} T^{3/2}}. \tag{4.2}$$

Here z is the ionic charge, the superscript* designates quantities relative to r_{min}, for example, $r_D^* = r_D / r_{min}$.

Now let us give further consideration to the case when during the collision process, the magnetic field exerts a significant influence on the motion of at least one of the interacting particles. The expressions for the viscosity coefficients become more complicated and with logarithmic accuracy, have the following form:

$$\eta_e^{(0)} = \frac{5 n_e T}{4 \nu_e} \left[(1 + R) \ln \rho_e^* + \frac{45}{16} R \ln \frac{r_D}{\rho_e} \right]^{-1} \quad (\rho_e \ll r_D) \tag{4.3}$$

$$\eta_i^{(0)} = \frac{5 n_i T}{4 \nu_i} \left[\ln \rho_i^* + \frac{45}{16} \ln \frac{r_D}{\rho_e} \right]^{-1} \qquad (\rho_i \ll r_D). \tag{4.4}$$

We shall calculate the mean energy of random motion (temperature) of each of the plasma components:

$$\frac{3}{2} n_\alpha T_\alpha = \frac{1}{2} \int f_\alpha^{(0)} m_\alpha v_\alpha^2 dv_\alpha = \frac{3}{2} n_\alpha T - \frac{1}{2} n_\alpha T (b_\alpha^{(0)} + 2b_\alpha^{(5)}) h_\mu h_\nu e_{\mu\nu}^\circ. \tag{4.5}$$

With (2.13) in mind, we come to the conclusion that the temperature of both plasma components in a weak magnetic feild is the same.

In a strong magnetic field the relation (2.13) no longer holds, and consequently, in agreement with (4.5), the electron temperature will not be equal to the ion temperature:

$$T_e - T_i = \frac{5 T \Lambda_0 h_\mu h_\nu e_{\mu\nu}^\circ}{12 \nu_i \Lambda^{(i)} (\ln \rho_e^* + \Lambda_0)} \tag{4.6}$$

At the same time, the pressure of the plasma components also changes (see (3.5)). The following notation has been introduced in equation (4.6):

$$\Lambda^{(i)} = \ln r_D^* \qquad (\rho_i \gg r_D), \tag{4.7}$$

$$\Lambda^{(i)} = \ln \rho_i^* + \frac{45}{16} \ln \frac{r_D}{\rho_i} \qquad (r_D \ll \rho_i), \tag{4.8}$$

$$\Lambda_0 = \frac{1}{2} \ln \frac{m_i}{m_e} \ln \frac{r_D}{\rho_e} \qquad \left(\rho_i \gg r_D \gg \rho_e \gg r_0 = \frac{m_i}{m_e} r_{min} \right), \tag{4.9}$$

$$\Lambda_0 = \frac{1}{4} \ln \frac{r_0}{\rho_e} \ln \rho_e^* r_0^* + \frac{1}{2} \ln \frac{m_i}{m_e} \ln \frac{r_D}{r_0}$$

$$(\rho_i \gg r_D \gg r_0 \gg \rho_e), \tag{4.10}$$

$$\Lambda_0 = \frac{1}{4} \ln \frac{r_D}{\rho_e} \ln \rho_e^* r_D^* \qquad (\rho_i \gg r_0 \gg r_D \gg \rho_e), \quad (4.11)$$

$$\Lambda_0 = \frac{1}{2} \ln \frac{\rho_i}{\rho_e} \ln \frac{m_i}{m_e} + \frac{1}{2} \ln \frac{r_D}{\rho_i} \ln \frac{m_i \rho_i}{m_e r_D}$$

$$\left(\frac{\sqrt{m_i}}{\sqrt{m_e}} \rho_i \gg r_D \gg \rho_i \gg \rho_e \gg r_0 \right) \qquad (4.12)$$

$$\Lambda_0 = \frac{1}{2} \ln \frac{\rho_i}{\rho_e} \ln \frac{m_i}{m_e} + \frac{1}{8} \ln^2 \frac{m_i \rho_i}{m_e r_D}$$

$$\left(r_D \gg \frac{\sqrt{m_i}}{\sqrt{m_e}} \rho_i \gg \rho_i \gg \rho_e \gg r_0 \right), \qquad (4.13)$$

$$\Lambda_0 = \frac{1}{4} \ln \frac{r_0}{\rho_e} \ln \rho_e^* r_0^* + \frac{1}{2} \ln \frac{m_i}{m_e} \ln \frac{\rho_i}{r_0} +$$

$$+ \frac{1}{2} \ln \frac{r_D}{\rho_i} \ln \frac{m_i \rho_i}{m_e r_D} \qquad \left(\frac{\sqrt{m_i}}{\sqrt{m_e}} \rho_i \gg r_D \gg \rho_i \gg r_0 \gg \rho_e \right), \quad (4.14)$$

$$\Lambda_0 = \frac{1}{4} \ln \frac{r_0}{\rho_e} \ln \rho_e^* r_0^* + \frac{1}{2} \ln \frac{m_i}{m_e} \ln \frac{\rho_i}{r_0} +$$

$$+ \frac{1}{8} \ln^2 \frac{m_i \rho_i}{m_e r_D} \qquad \left(r_D \gg \frac{\sqrt{m_i}}{\sqrt{m_e}} \rho_i \gg \rho_i \gg r_0 \gg \rho_e \right), \quad (4.15)$$

$$\Lambda_0 = \frac{1}{4} \ln \frac{r_D}{\rho_e} \ln \rho_e^* r_D^* \begin{cases} r_0^{1/3} \rho_i^{2/3} \equiv r_1 \gg \rho_i \gg r_D \gg \rho_i \\ r_1 \gg r_D \gg \rho_i \gg \rho_e \end{cases} \quad (4.16)$$

$$\Lambda_0 = \frac{1}{4} \ln \frac{r_1}{\rho_e} \ln r_1^* \rho_e^* + \ln \frac{r_D}{r_1} \ln \left| \frac{\Omega_e \rho_e}{\Omega_i \sqrt{r_D r_1}} \right|$$

$$\left(\rho_e \left| \frac{\Omega_e}{\Omega_i} \right| \gg r_D \gg r_1 \gg \rho_i \gg \rho_e \right), \qquad (4.17)$$

$$\Lambda_0 = \frac{1}{4} \ln \frac{r_1}{\rho_e} \ln r_1^* \rho_e^* + \frac{1}{2} \ln^2 \left| \frac{\rho_e}{r_1} \frac{\Omega_e}{\Omega_i} \right|$$

$$\left(r_D \gg \rho_e \left| \frac{\Omega_e}{\Omega_i} \right| \gg r_1 \gg \rho_i \gg \rho_e \right). \qquad (4.18)$$

b) Coefficients $\eta_\alpha^{(2)}$ and $\eta_\alpha^{(4)}$. The coefficient $\eta_\alpha^{(4)}$ does not depend on the collision frequency in the first approximation:

$$\eta_\alpha^{(4)} = -\frac{n_\alpha T}{\Omega_\alpha}.$$

For the coefficient $\eta_\alpha^{(2)}$ we obtain

$$\eta_e^{(2)} = \frac{4 n_e T \nu_e}{\Omega_e^2} \left(\frac{1}{2} + \frac{3}{10} R \right) \ln r_D^* \qquad (\rho_e \gg r_D), \quad (4.19)$$

$$\eta_i^{(2)} = \frac{6 n_i T \nu_i}{5 \Omega_i^2} \ln r_D^* \qquad (\rho_i \gg r_D). \quad (4.20)$$

For stronger fields we have

$$\eta_e^{(2)} = \frac{4 n_e T \nu_e}{\Omega_e^2} \left(\frac{1}{2} + \frac{3}{10} R \right) \ln \rho_e^* + \frac{3 n_e T \nu_e}{4 \Omega_e^2} \Lambda_1, \quad (4.21)$$

where Λ_1, depending on the magnetic field strength, and the density and temperature of the plasma, has the following form:

$$\Lambda_1 = \ln \frac{m_i}{m_e} \ln \frac{r_D}{\rho_e} \qquad (\rho_i \gg r_D \gg \rho_e \gg r_0),$$

$$\Lambda_1 = \ln \frac{m_i}{m_e} \ln \frac{r_D}{\rho_e} + \frac{1}{2} \ln \frac{r_0}{\rho_e} \ln r_D^* \rho_e^* \qquad (\rho_i \gg r_D \gg r_0 \gg \rho_e),$$

$$\Lambda_1 = \frac{1}{2} \ln \frac{r_D}{\rho_e} \ln r_D^* \rho_e^* \qquad (\rho_i, r_0 \gg r_D \gg \rho_e),$$

$$\Lambda_1 = \frac{1}{2} \ln^2 \frac{m_i}{m_e} + \frac{1}{2} \ln \frac{r_D}{\rho_i} \ln r_D^* \rho_i^* \qquad (r_D \gg \rho_i \gg \rho_e \gg r_0),$$

$$\Lambda_1 = \ln \frac{m_i}{m_e} \ln \frac{\rho_i}{r_0} + \frac{1}{2} \ln \frac{r_0}{\rho_e} \ln \rho_e^* r_0^* + \frac{1}{2} \ln \frac{r_D}{\rho_i} \ln r_D^* \rho_i^*,$$

$$(r_D \gg \rho_i \gg r_0 \gg \rho_e),$$

$$\Lambda_1 = \frac{1}{4} \ln \frac{m_i}{m_e} \ln r_D^* \rho_e^* + \frac{1}{2} \ln \frac{r_D}{\rho_i} \ln r_D^* \rho_i^*,$$

$$(r_D, r_0 \gg \rho_i \gg \rho_e). \qquad (4.22)$$

Ion–ion collisions make the main contribution to the viscosity coefficient $\eta_i^{(2)}$. In the case when the ion Larmor radius becomes less than r_D we obtain, instead of formula (4.20),

$$\eta_i^{(2)} = \frac{6 \nu_i n_i T}{5 \Omega_i^2} \ln \rho_i^*. \qquad (4.23)$$

c) Coefficients $\eta_\alpha^{(1)}$ and $\eta_\alpha^{(3)}$. The coefficient $\eta_\alpha^{(3)}$, as in [4], does not depend on the collision frequency:

$$\eta_\alpha^{(3)} = -\frac{n_\alpha T}{2 \Omega_\alpha}. \qquad (4.24)$$

For the coefficient $\eta_\alpha^{(1)}$ we obtain

$$\eta_\alpha^{(1)} = \frac{1}{4} \eta_\alpha^{(2)} \qquad (\rho_e \gg r_D). \qquad (4.25)$$

For stronger magnetic fields we have

$$\eta_e^{(1)} = \frac{\nu_e}{\Omega_e^2} \ln \rho_e^* \left[\frac{1}{2} + \frac{3}{10} R + \frac{1}{4} R \left(\frac{\pi}{2} + \sqrt{2\pi} \right) \right]$$

$$(\rho_e \ll r_D) \qquad (4.26)$$

$$\eta_i^{(1)} = \frac{\nu_i}{\Omega_i^2} \ln \rho_i^* \left[\frac{3}{10} + \frac{1}{4} \left(\frac{\pi}{2} + \sqrt{2\pi} \right) \right] \quad (\rho_i \ll r_D), \quad (4.27)$$

A few words should be said about the values of quantities conditioned by the influence of a strong magnetic field on particle collisions. We shall consider, for example, the log-log contributions represented by formulas (4.22). We note that the first three formulas of this group hold when $\rho_e \ll r_D \ll \rho_i$, i.e., for a hydrogen plasma on fulfillment of the conditions

$$1 \ll 4.1 \cdot 10^2 H / \sqrt{n} \ll 43. \qquad (4.28)$$

Here H is the magnetic field in gauss, n is the density. The last three formulas from group (4.22) are valid on condition that

$$\rho_e \ll \rho_i \ll r_D,$$

i.e.,

$$H/\sqrt{n} \gg 0.1.$$

The plasma temperature determines the quantity $r_0 = 3 \cdot 10^{-4} \cdot T^{-1}$, where T is the temperature in electron volts, and thus distinguishes different particular cases from (4.22).

As an example, we shall consider a plasma with density $n \approx 10^{10}$ cm^{-3}, situated in a magnetic field of field strength $H \approx 10^4$ gauss at a temperature of 1 eV.
In this case $r_D \approx 0.7 \cdot 10^{-2}$ cm, $\rho_e \approx 0.17 \cdot 10^{-3}$ cm, $r_{min} \approx 0.74 \cdot 10^{-6}$ cm and $\Lambda_1 \approx 28.0$, while the Coulomb log equals 10.6. Hence it is evident that the corrections due to taking the influence of the magnetic field on particle collisions into account exceed the Coulomb log obtained in the usual theory by a factor of more than two.

It is clear from the expressions cited above for the viscosity coefficients that the plasma viscosity manifests itself in a substantially different way in a strong magnetic field that affects particle collisions. For example, a temperature difference develops between the electron and ion plasma components proportional to the drift velocity $h_\mu h_\nu e_{\mu\nu}^\circ$ and depending logarithmically on the magnetic field (4.6).

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REFERENCES

1. S. Chapman and T. Cowling, Mathematical Theory of Non-Uniform Gases [Russian translation], Izd-vo inostr. lit., 1960.
2. W. Marshall, "Kinetic theory of ionized gases," AERE, t/r 2419, 1960.
3. J. Wright, Phys. Fluids, vol. 4, p. 11, 1961.
4. S. I. Braginskii, Problems of Plasma Theory [in Russian], Gosatomizdat, 1963.
5. V. P. Silin and L. M. Gorbunov, AN SSSR, vol. 145, p. 1265, 1962.
6. V. P. Silin, Yademyi Sintez, vol. 2, no. 3, 4, 1962.
7. S. T.Belyaev, Plasma Physics and the Problem of Controlled Thermonuclear Reactions [in Russian], Izd-vo AN SSSR, vol. 3, p. 66, 1958.
8. V. P. Silin, Zh. eksperim. i teor. fiz. vol. 38, p. 1771, 1960.
9. Yu. M. Aliev and V. P. Silin, "On the rate of equalization of 'longitudinal' and 'transverse' plasma temperatures in a magnetic field," Yademyi sintez, no. 3, 1963.
10. Yu. M. Aliev and A. R. Shister, "Transport phenomenon in a plasma in a strong magnetic field," ZhETF, vol. 45, no. 11, p. 1499, 1963.

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